An introduction to Information Theory and some of its applications: Black Hole Information Paradox and Renormalization Group Information Flow

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Classical Information Theory (CIT) was devised to quantify the elusive concept of *information*. I introduce its basic definitions and some of its formal tools: the *information content* of a random variable, the *typical set* of an array of independent, identically distributed random variables, and some basic principles of *data compression*. I then describe the evolution of CIT into Quantum Information Theory (QIT), introducing concepts such as the *qubit*, the *pure* and *mixed states*, the *Holevo bound*, the *no-cloning theorem*, and the *quantum complementarity*, a concept that is exploited by some protocols of Quantum Cryptography. I then illustrate two examples of how the tools of QIT can be applied to other fields of physics.

First I describe the Black Hole (BH) Information Paradox, a theoretical problem related to the phenomenon of the Hawking radiation. Considering a BH starting in a pure state, after its complete evaporation only the Hawking radiation remains, which is shown to be in a mixed state with no correlation with the initial state. This either describes a non-unitary evolution of an isolated system, contradicting the evolution postulate of Quantum Mechanics, or it implies that the initial information content can escape the BH, therefore contradicting its description given by General Relativity. It can also shown that the escape of information from a BH also violates the *no-cloning theorem*. I also describe some progress done toward the solution of the paradox.

As a second example I describe is the Flow of Information along the Renormalization Group (RG). The RG is a tool developed at first to Quantum Field Theory, and then to other fields, such as Solid State Physics, Statistical Mechanics, and Cosmology. The c-theorem proves that a function exist, which is monotonically decreasing along the RG transformation, at least in the case of 2D systems. Because of this monotonicity, and the consequent irreversibility of the group transformation, some authors suggest an interpretation in terms of *information flow* for the c-function.